2226 – 5220M

Diploma Programme
Programme du diplôme
Programa del Diploma

Markscheme

May 2026

Mathematics: applications and interpretation

Higher level

Paper 1

24 pages

Instructions to Examiners

Abbreviations

M Marks awarded for attempting to use a correct **Method**.

A Marks awarded for an **Answer** or for **Accuracy**; often dependent on preceding ***M*** marks.

R Marks awarded for clear **Reasoning**.

AG Answer given in the question and so no marks are awarded.

FT Follow through. The practice of awarding marks, despite student errors in previous parts, for their correct methods/answers using incorrect results.

Using the markscheme

1 General

Award marks using the annotations as noted in the markscheme *eg **M1**, **A2**.*

2 Method and Answer/Accuracy marks

- Do **not** automatically award full marks for a correct answer; all working **must** be checked, and marks awarded according to the markscheme.
- It is generally not possible to award ***M0*** followed by ***A1***, as ***A*** mark(s) depend on the preceding ***M*** mark(s), if any.
- Where ***M*** and ***A*** marks are noted on the same line, *e.g.* ***M1A1***, this usually means ***M1*** for an **attempt** to use an appropriate method (*e.g.* substitution into a formula) and ***A1*** for using the **correct** values.
- Where there are two or more ***A*** marks on the same line, they may be awarded independently; so if the first value is incorrect, but the next two are correct, award ***A0A1A1***.
- Where the markscheme specifies ***A3***, ***M2*** *etc.*, do **not** split the marks, unless there is a note.
- The response to a "show that" question does not need to restate the ***AG*** line, unless a **Note** makes this explicit in the markscheme.
- Once a correct answer to a question or part question is seen, ignore further working even if this working is incorrect and/or suggests a misunderstanding of the question. This will encourage a uniform approach to marking, with less examiner discretion. Although some students may be advantaged for that specific question item, it is likely that these students will lose marks elsewhere too.
- An exception to the previous rule is when an incorrect answer from further working is used **in a subsequent part**. For example, when a correct exact value is followed by an incorrect decimal approximation in the first part and this approximation is then used in the second part. In this situation, award ***FT*** marks as appropriate but do not award the final ***A1*** in the first part.

Examples:

	Correct answer seen	**Further working seen**	**Any FT issues?**	**Action**
1.	$8\sqrt{2}$	5.65685... *(incorrect decimal value)*	No. Last part in question.	Award ***A1*** for the final mark *(condone the incorrect further working)*
2.	$\frac{35}{72}$	0.468111… (incorrect decimal value)	Yes. Value is used in subsequent parts.	Award ***A0*** for the final mark *(and full **FT** is available in subsequent parts)*

3 Implied marks

Implied marks appear in **brackets e.g.** ***(M1)***,and can only be awarded if **correct** work is seen or implied by subsequent working/answer.

4 Follow through marks (only applied after an error is made)

Follow through (***FT***) marks are awarded where an incorrect answer from one **part** of a question is used correctly in **subsequent** part(s) (e.g. incorrect value from part (a) used in part (d) or incorrect value from part (c)(i) used in part (c)(ii)). Usually, to award ***FT*** marks, **there must be working present** and not just a final answer based on an incorrect answer to a previous part. However, if all the marks awarded in a subsequent part are for the answer or are implied, then ***FT*** marks should be awarded for *their* correct answer, even when working is not present.

For example: following an incorrect answer to part (a) that is used in subsequent parts, where the markscheme for the subsequent part is ***(M1)A1***, it is possible to award full marks for *their* correct answer, **without working being seen.** For longer questions where all but the answer marks are implied this rule applies but may be overwritten by a **Note** in the Markscheme.

- Within a question part, once an **error** is made, no further ***A*** marks can be awarded for work which uses the error, but ***M*** marks may be awarded if appropriate.

- If the question becomes much simpler because of an error then use discretion to award fewer ***FT*** marks, by reflecting on what each mark is for and how that maps to the simplified version.

- If the error leads to an inappropriate value (*e.g.* probability greater than 1, $\sin\theta = 1.5$, non-integer value where integer required), do not award the mark(s) for the final answer(s).

- The markscheme may use the word “their” in a description, to indicate that students may be using an incorrect value.

- If the student’s answer to the initial question clearly contradicts information given in the question, it is not appropriate to award any ***FT*** marks in the subsequent parts. This includes when students fail to complete a “show that” question correctly, and then in subsequent parts use their incorrect answer rather than the given value.

- Exceptions to these ***FT*** rules will be explicitly noted on the markscheme.

- If a student makes an error in one part but gets the correct answer(s) to subsequent part(s), award marks as appropriate, unless the command term was “Hence”.

5 Mis-read

If a student incorrectly copies values or information from the question, this is a mis-read (***MR***). A student should be penalized only once for a particular misread. Use the ***MR*** stamp to indicate that this has been a misread and do not award the first mark, even if this is an ***M*** mark, but award all others as appropriate.

- If the question becomes much simpler because of the ***MR***, then use discretion to award fewer marks.
- If the ***MR*** leads to an inappropriate value (*e.g.* probability greater than 1, $\sin\theta = 1.5$, non-integer value where integer required), do not award the mark(s) for the final answer(s).
- Miscopying of students' own work does **not** constitute a misread, it is an error.
- If a student uses a correct answer, to a "show that" question, to a higher degree of accuracy than given in the question, this is NOT a misread and full marks may be scored in the subsequent part.
- ***MR*** can only be applied when work is seen. For calculator questions with no working and incorrect answers, examiners should **not** infer that values were read incorrectly.

6 Alternative methods

Students will sometimes use methods other than those in the markscheme. Unless the question specifies a method, other correct methods should be marked in line with the markscheme. If the command term is 'Hence' and not 'Hence or otherwise' then alternative methods are not permitted unless covered by a note in the mark scheme.

- Alternative methods for complete questions are indicated by **METHOD 1**, **METHOD 2**, *etc*.
- Alternative solutions for parts of questions are indicated by **EITHER** . . . **OR**.

7 Alternative forms

Unless the question specifies otherwise, **accept** equivalent forms.

- As this is an international examination, accept all alternative forms of **notation** for example 1.9 and 1,9 or 1000 and 1,000 and 1.000.
- Do not accept final answers written using calculator notation. However, ***M*** marks and intermediate ***A*** marks can be scored, when presented using calculator notation, provided the evidence clearly reflects the demand of the mark.
- In the markscheme, equivalent **numerical** and **algebraic** forms will generally be written in brackets immediately following the answer.
- In the markscheme, some **equivalent** answers will generally appear in brackets. Not all equivalent notations/answers/methods will be presented in the markscheme and examiners are asked to apply appropriate discretion to judge if the student work is equivalent.

8 Format and accuracy of answers

If the level of accuracy is specified in the question, a mark will be linked to giving the answer to the required accuracy. If the level of accuracy is not stated in the question, the general rule applies to final answers: *unless otherwise stated in the question all numerical answers must be given exactly or correct to three significant figures.*

Where values are used in subsequent parts, the markscheme will generally use the exact value, however students may also use the correct answer to 3 sf in subsequent parts. The markscheme will often explicitly include the subsequent values that come "*from the use of 3 sf values*".

Simplification of final answers: Students are advised to give final answers using good mathematical form. In general, for an ***A*** mark to be awarded, arithmetic should be completed, and any values that lead to integers should be simplified; for example, $\sqrt{\frac{25}{4}}$ should be written as $\frac{5}{2}$. An exception to this is simplifying fractions, where lowest form is not required (although the numerator and the denominator must be integers); for example, $\frac{10}{4}$ may be left in this form or written as $\frac{5}{2}$. However, $\frac{10}{5}$ should be written as 2, as it simplifies to an integer.

Algebraic expressions should be simplified by completing any operations such as addition and multiplication, e.g. $4e^{2x} \times e^{3x}$ should be simplified to $4e^{5x}$, and $4e^{2x} \times e^{3x} - e^{4x} \times e^{x}$ should be simplified to $3e^{5x}$. Unless specified in the question, expressions do not need to be factorized, nor do factorized expressions need to be expanded, so $x(x+1)$ and $x^2 + x$ are both acceptable.

Please note: intermediate ***A*** marks do NOT need to be simplified.

9 Calculators

A GDC is required for this paper, but if you see work that suggests a student has used any calculator not approved for IB DP examinations (eg CAS enabled devices), please follow the procedures for malpractice.

10 Presentation of student work

Crossed out work: If a student has drawn a line through work on their examination script, or in some other way crossed out their work, do not award any marks for that work unless an explicit note from the student indicates that they would like the work to be marked.

More than one solution: Where a student offers two or more different answers to the same question, an examiner should only mark the first response unless the student indicates otherwise. If the layout of the responses makes it difficult to judge, examiners should apply appropriate discretion to judge which is "first".

1. (a) $0.08 \left(\frac{16}{200}, \frac{2}{25}\right)$ ***A1***

[1 mark]

(b) $0.727 \left(\frac{16}{22}, \frac{8}{11}, 0.727272...\right)$ ***A1A1***

Note: ***A1*** for correct numerator and ***A1*** for correct denominator.

[2 marks]

(c) **METHOD 1**

Valid attempt to find the probability of one or two left-handed, award ***M1*** for any one of $\frac{22}{200}\times\frac{178}{199}$ or $\frac{178}{200}\times\frac{22}{199}$ or $\frac{22}{200}\times\frac{21}{199}$ seen. ***M1***

$\left(\frac{22}{200}\times\frac{178}{199}\right)+\left(\frac{178}{200}\times\frac{22}{199}\right)+\left(\frac{22}{200}\times\frac{21}{199}\right)$ ***A1***

$0.208 \left(0.208391..., \frac{8294}{39800}, \frac{4147}{19900}\right)$ ***A1***

METHOD 2

Valid attempt to use the probability of no left-handed ***M1***

$1-\left(\frac{178}{200}\times\frac{177}{199}\right)$ ***A1***

$0.208 \left(0.208391..., \frac{8294}{39800}, \frac{4147}{19900}\right)$ ***A1***

Note: Check working because invalid methods can lead to an answer of 0.208

Award ***M0A0A0*** for:

$1-\left(\frac{178}{200}\right)^2 = 0.208$

OR

$X \sim \mathrm{B}\left(2, \frac{22}{200}\right)$ $\mathrm{P}(X \geq 1) = 1 - \mathrm{P}(X = 0) = 0.208$

Award ***M0A0A1*** for a correct answer of 0.208 with NO WORKING.

[3 marks]

Total [6 marks]

2. (a) attempt to calculate constant of proportionality ***(M1)***

$187.50 = \frac{k}{4}$ OR 4×187.5 OR 750 OR $c \propto \frac{1}{n}$ or $c = \frac{k}{n}$ seen

$c = \frac{750}{n}$ ***A1***

[2 marks]

(b) (i) $\frac{750}{a} < 100$ ***(M1)***

$a > 7.5$

$(a =) 8$ ***A1***

[2 marks]

(ii) $c = \frac{750}{8}$ or use of technology ***(M1)***

$(c =) 93.75 \text{ USD}$ ***A1***

Note: Accept 93.8 USD. Do not award ***FT*** for an answer greater than or equal to 100 USD. USD not required.

[2 marks]

Total [6 marks]

3. (a) $299 \times 218 \times 318$ ***(M1)***

$20\,727\,876\,\text{mm}^3$ ***A1***

[2 marks]

(b) attempt to convert to litres ***M1***

$20\,727\,876\,\text{mm}^3 \times \frac{0.001\,\text{mL}}{1\,\text{mm}^3} \times \frac{1\text{L}}{1000\,\text{mL}}$

Note: Award ***M1*** for attempt to convert to litres or mm^3 or mL.

20.727876 litres or $20727.876\,\text{mL}$ seen ***A1***

attempt to use percentage error formula ***(M1)***

$\left|\frac{20.727876 - 20}{20}\right| \times 100$ or $\left|\frac{20\,727.876 - 20\,000}{20\,000}\right| \times 100$

Note: Award the ***M1*** for substitution of either part (a) or their converted value i.e. ft within this part; the denominator must be 20 (if working in L) or $20\,000$ (if working in mL).

$3.64\ (3.63938...)\%$ ***A1***

Note: Award ***A3*** for an answer of 3.64% with no working

[4 marks]

(c) Any reasonable explanation: ***R1***

the cutouts in the actual carboy make his calculated volume greater

the carboy has round edges so it is not a cuboid

the stated volume may be an internal volume instead of an external volume

the stated dimensions could be external dimensions rather than internal

Note: Award ***R0*** for an explanation based on the mechanics of Shaun's calculations such as "the store might round the information up to the nearest length, width & height, therefore Shaun's calculation might overestimate the volume".

[1 mark]

Total [7 marks]

4. (a) $2.8 = a + b + c$ (or $2.8 = a(1)^2 + b(1) + c$)

$2.86 = 4a + 2b + c$ (or $2.86 = a(2)^2 + b(2) + c$)

$2.89 = 9a + 3b + c$ (or $2.89 = a(3)^2 + b(3) + c$) ***A2***

Note: Award ***A1A0*** for one correct or two correct.
Award ***A1A0*** for x-coordinates substituted in but not the y- coordinates

$h(x) = a(1)^2 + b(1) + c$

$h(x) = a(2)^2 + b(2) + c$

$h(x) = a(3)^2 + b(3) + c$

Award ***A0A0*** for use of QuadReg in part (a)

[2 marks]

(b) $h(x) = -0.015x^2 + 0.105x + 2.71$ or $h(x) = -\frac{3}{200}x^2 + \frac{21}{200}x + \frac{271}{100}$ ***A2***

[2 marks]

(c) the height of the ball at the moment Emily hits the ball ***R1***

Note: Award ***R0*** for
- Emily's height or
- c is the starting point of the serve or
- y-intercept of the equation or
- the height of the ball **before** Emily hits it

[1 mark]

(d) evidence of finding the height when $x = 9$ ***M1***

$(h(9) =) 2.44 > 2.24$ ***A1***

Note: Award ***M1A1*** for their $h(9)$ or finding their x for $h(x) = 2.24$.

evidence of either finding the x-intercept or the height when $x = 18$ ***M1***

$(x =) 17.3894... < 18$ **OR** $(h(18) =) -0.26 < 0$ ***A1***

Note: Award ***M1A1*** for their x for $h(x) = 0$ OR their $h(18)$.

therefore, the ball is in play since it has gone over the net and hit the court before crossing the baseline ***A1***

Note: The conclusion could be worded differently.
For example:
The ball hit the court on the other side, therefore Emily's score will be in play.
Emily's serve is in play since it goes over the net and bounces before the endline.

Note: Award ***A1FT*** for a correct conclusion from their data.

Note: The inequalities can also be provided in words.

[5 marks]

Total [10 marks]

5. (a) $f'(6) = -1.5$ ***(A1)***

gradient of normal is $\frac{2}{3}$ $(0.666666..., 0.667)$ ***A1***

Note: ***FT*** from their $f'(6)$ so can award ***A0A1FT***.

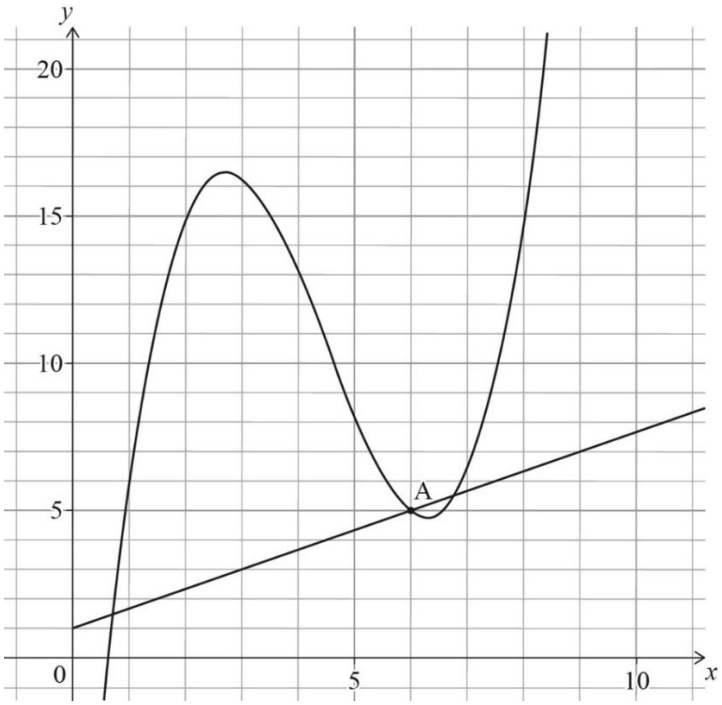
[2 marks]

(b) $y - 5 = \frac{2}{3}(x - 6)$ or equivalent e.g. $y = \frac{2}{3}x + 1$ ***A1***

Note: Allow ***FT*** from (a) providing a gradient of the normal has been found in (a)
If there is no attempt to find the gradient of the normal and they use their gradient of the tangent then award ***A1DM*** for $y - 5 = m(x - 6)$ where *m* is the gradient they found for $f(x)$.

[1 mark]

(c)



straight line (or a ray) drawn which passes through A. ***A1***

Note: Award ***A1FT*** for any straight line through A that uses their normal gradient from part (a)
It should not approximate to a tangent.

crosses y-axis at $y = 1$ ***A1***

Note: Award ***A1FT*** if the line looks like it is in an approximately normal direction to the curve at A.

[2 marks]

continued…

Question 5 continued

(d) $(6.82, 5.54)$ $((6.81526..., 5.54350...))$ ***A1***

$(0.685, 1.46)$ $((0.684737..., 1.45649...))$ ***A1***

Note: If ***FT*** from (a) and (b) award ***A1FT*** for correct coordinates of both points of intersection. (OR ***A1FTA0*** if there is only one).

[2 marks]

(e) find corresponding value for x. ***M1***

$x = 2$

then $g(x) = -1$

(the point is) $(2, -1)$ ***A1***

[2 marks]

Total [9 marks]

6. (a) 354° ***A1***

[1 mark]

(b) recognition of isosceles triangle ***(M1)***

$\frac{180-6}{2}$

87° ***A1***

[2 marks]

(c) use of cosine rule ***(M1)***

$x^2 = 5^2 + 5^2 - 2(5)(5)\cos 6$ ***(A1)***

$x = 0.523\ (0.5233595...)$ km ***A1***

OR

use of sine rule ***(M1)***

$\frac{\sin 6^\circ}{x} = \frac{\sin 87^\circ}{5}$ ***(A1)***

$x = 0.523\ (0.5233595...)$ km ***A1***

OR

use of isosceles triangle ***(M1)***

$\sin 3^\circ = \frac{0.5x}{5}$ OR $\cos 87^\circ = \frac{0.5\text{CB}}{5}$ ***(A1)***

$x = 0.523\ (0.5233595...)$ km ***A1***

[3 marks]

(d) $\frac{5 + 0.5233595...}{2.25}$ ***(M1)***

(average speed) = $2.45\ (2.45482...)$ km/h ***A1***

[2 marks]

Total [8 marks]

7. (a) **METHOD 1**

evidence of using the inverse of matrix T ***(M1)***

$C_{\text{last month}} = T^{-1}C \qquad (T^{-1} = \begin{pmatrix} 1.4 & -0.6 \\ -0.4 & 1.6 \end{pmatrix})$ ***(A1)***

$\begin{pmatrix} 0.7 \\ 0.3 \end{pmatrix}$ **OR** 70% ordered regular coffee and 30% ordered decaf ***A1***

METHOD 2

evidence of using simultaneous equation ***(M1)***

$0.8r + 0.3d = 0.65$
$0.2r + 0.7d = 0.35$ ***(A1)***

$r = 0.7 \quad d = 0.3$ **OR** 70% ordered regular coffee and 30% ordered decaf ***A1***

[3 marks]

(b) **METHOD 1**

Attempt to use $Tx = x$ ***M1***

$\begin{pmatrix} 0.8 & 0.3 \\ 0.2 & 0.7 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$ **OR** $(0.8x_1 + 0.3x_2 = x_1$ **AND** $0.2x_1 + 0.7x_2 = x_2)$ ***A1***

$x_1 = \frac{3}{2}x_2$ **OR** $x_2 = \frac{2}{3}x_1$ seen ***A1***

$x_1 + x_2 = 1$ OR scaling by $\frac{1}{x_1 + x_2}$ seen ***A1***

$x_1 = 0.6,\ x_2 = 0.4$ ***AG***

METHOD 2

attempt to find matrix T in the form PDP^{-1} ***M1***

$T = \begin{pmatrix} 1 & 3 \\ -1 & 2 \end{pmatrix}\begin{pmatrix} 0.5 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 3 \\ -1 & 2 \end{pmatrix}^{-1}$ ***A1***

attempt to find T^{∞} ***M1***

$T^{\infty} = \begin{pmatrix} 1 & 3 \\ -1 & 2 \end{pmatrix}\begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 3 \\ -1 & 2 \end{pmatrix}^{-1}$

$T^{\infty} = \begin{pmatrix} 0.6 & 0.6 \\ 0.4 & 0.4 \end{pmatrix}$ ***A1***

$x_1 = 0.6,\ x_2 = 0.4$ ***AG***

continued…

Question 7 continued

METHOD 3

Attempt to find T^{∞} using T^{n} where n is very large ***M1***

finding. $\begin{pmatrix} 0.8 & 0.3 \\ 0.2 & 0.7 \end{pmatrix}^{n}$ **OR** $\begin{pmatrix} 0.8 & 0.3 \\ 0.2 & 0.7 \end{pmatrix}^{n}\begin{pmatrix} 0.65 \\ 0.35 \end{pmatrix}$ for a suitably large value of n $(n>10)$ ***M1***

$T^{\infty}=\begin{pmatrix} 0.6 & 0.6 \\ 0.4 & 0.4 \end{pmatrix}$ **OR** $T^{\infty}\begin{pmatrix} 0.65 \\ 0.35 \end{pmatrix}=\begin{pmatrix} 0.6 \\ 0.4 \end{pmatrix}$ ***A1***

The steady state vector is $\begin{pmatrix} 0.6 \\ 0.4 \end{pmatrix}$ ***AG***

$\begin{pmatrix} 0.8 & 0.3 \\ 0.2 & 0.7 \end{pmatrix}\begin{pmatrix} 0.6 \\ 0.4 \end{pmatrix}=\begin{pmatrix} 0.6 \\ 0.4 \end{pmatrix}$ ***A1***

Since $\begin{pmatrix} 0.6 \\ 0.4 \end{pmatrix}$ is invariant under T it is the only steady state vector ***R1***

Note: Do not award ***M1A1A0R1***

[4 marks]

Total *[7 marks]*

8. (a) attempt to substitute y -coordinates into trapezoidal rule ***(M1)***

$\frac{1}{2}(2)\big((3.90+3.08)+2(1.63+1.48+3.07)\big)$ ***(A1)***

$19.3\ (19.34)\,\text{m}$ ***A1***

[3 marks]

(b) recognition to find the integral of $v(t)$ ***(M1)***

$\int_0^8\left(\sqrt{\frac{1}{2}(t-3)^2+2\cos(t)+3}\right)\text{d}t$ ***(A1)***

$19.0\ (18.9760...)\,\text{m}$ ***A1***

Note: Award ***M1A1A0*** for an unsupported answer of $22.4396...$ from use of degrees. Accept $19\ \text{m}$ as the final answer

[3 marks]

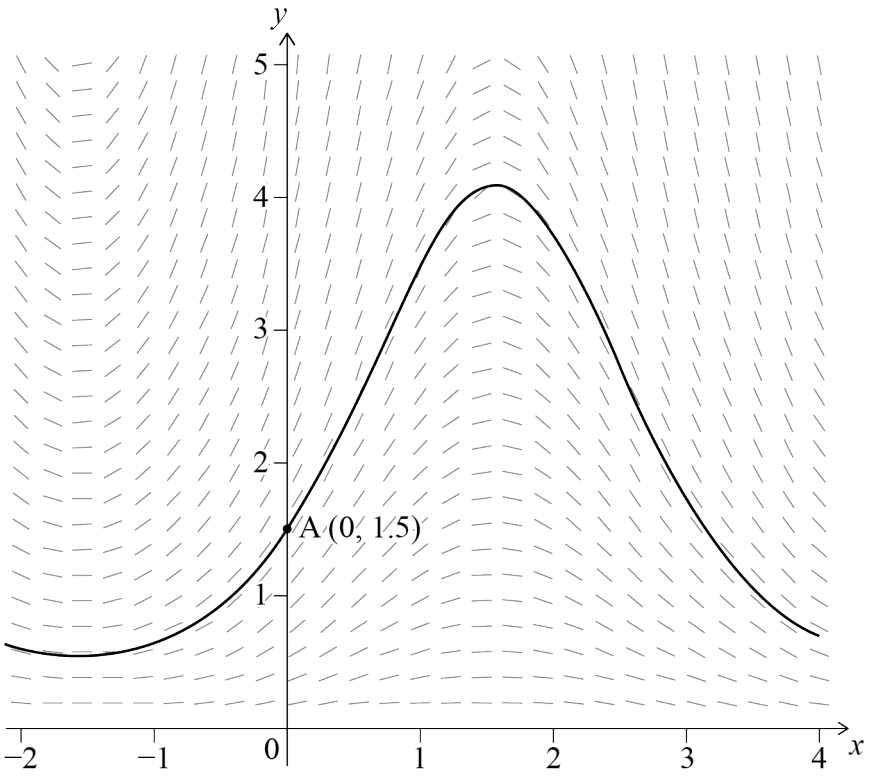
Total *[6 marks]*

9. (a) curve goes through the point $(0, 1.5)$ ***A1***

approximately correct shape with maximum occurring at $x = \frac{\pi}{2} \approx 1.5$

and between $y = 3.5$ and $y = 4.5$. ***A1***

[2 marks]



Note: A trajectory only showing positive x-values should be awarded at most ***A1A0***.

(b) attempt to separate variables ***(M1)***

$$\frac{1}{y}\mathrm{d}y = \cos x\,\mathrm{d}x$$

$$\int \frac{1}{y}\mathrm{d}y = \int \cos x\,\mathrm{d}x$$

$$\ln|y| = \sin(x) + c$$ ***(A1)(A1)***

Note: Condone $\ln y$ in place of $\ln|y|$ and condone missing + c

$$y = A\mathrm{e}^{\sin x}$$ ***(A1)***

$$y = 1.5\mathrm{e}^{\sin x}$$ ***A1***

[5 marks]

Total *[7 marks]*

10. (a) 150 mg/L ***A1***

[1 mark]

(b) **METHOD 1**

attempt to find the composite function $E(t)$ ***(M1)***

$E(t) = \ln\left(\frac{150}{1+0.35t^2}+1.5\right) - \frac{1}{\frac{150}{1+0.35t^2}+1}$ ***(A1)***

setting their $E(t)$ equal to 2 ***(M1)***

$2 = \ln\left(\frac{150}{1+0.35t^2}+1.5\right) - \frac{1}{\frac{150}{1+0.35t^2}+1}$

$(t =) 7.70\ (7.70484...)$ hours ***A1***

METHOD 2

attempt to solve $E = 2$ for C ***(M1)***

$\ln(C+1.5) - \frac{1}{C+1} = 2$

$C = 6.88779...$ ***(A1)***

substituting for C ***(M1)***

$\frac{150}{1+0.35t^2} = 6.88779...$

$(t =) 7.70\ (7.70484...)$ hours ***A1***

Note: Award at most ***M1A1M1A0*** if negative solution is given as an answer.
Condone 7.71 or 7.7 as a final answer

[4 marks]

Total *[5 marks]*

11. (a) **EITHER**

evidence of finding the natural log of the population data ***(M1)***

OR

evidence of the use of exponential regression in the form a*b^x (or a*e^bx ***(M1)***
$N(t) = 0.486442...(1.37379...)^t$ **OR** $N(t) = 0.486442...\left(e^{(0.317575...)t}\right)$

THEN

$\ln(N(t)) = 0.318t - 0.721$ $\left(\ln(N(t)) = 0.317575...t - 0.720636...\right)$ ***A1A1***

Note: Award ***M0A1A0*** for linear regression of N against t $0.667270...t - 0.289063...$
Logarithmic regression of t against N, can lead to a correct answer that should be marked as a VAM.
Condone $\ln(N(t)) = 0.318x - 0.721$

[3 marks]

(b) attempt to convert equation to new form ***(M1)***
$e^{\ln(N(t))} = e^{0.317575...t - 0.720636...}$

OR if exponential regression is used then $(1.37379...)^t = e^{\ln(1.37379...)t} = e^{(0.317575...)t}$

OR referring to the working from part (a) directly
$c = 0.486$ $(0.486442...)$, $d = 0.318$ $(0.317575...)$ ***A1A1***

Note: Award ***M1A1A0*** for exponential regression leading to $0.486442...(1.37379...)^x$

[3 marks]
Total *[6 marks]*

12. (a) **EITHER**

recognition to use reverse chain rule (i.e. moving 2 outside integral) or integration by substitution ***(M1)***

$\left(g\left(x\right)=\right)2\sin x^3+c$

OR

differentiating $a\sin x^n+b$ ***(M1)***

$\left(g'\left(x\right)=\right)a\cos x^n\cdot nx^{n-1}$

$a=2 \quad n=3$

THEN

substitute to find c ***(M1)***

$1=2\sin\left(0\right)^3+c$

$\left(g\left(x\right)=\right)2\sin x^3+1$ ***A1***

[3 marks]

(b) (i) attempt to find point of intersection ***(M1)***

$x=\sqrt[3]{\frac{\pi}{2}}\approx 1.16244...$ seen ***A1***

(ii) attempt to find the difference between two volumes of revolution ***(M1)***

$\pi\left(3\right)^2\left(1.16244...\right)-\int_0^{1.16244...}\pi\left(2\sin x^3+1\right)^2\,\mathrm{d}x$ ***A1***

(volume=) 21.0 (21.0334 ...) ***A1***

Note: Accept 21 for a final answer

[5 marks]

Total [8 marks]

13. (a) attempt to find modulus of z ***(M1)***

$\sqrt{(-4)^2+\left(\sqrt{6}-\sqrt{2}\right)^2}$

correct expansion of $\left(\sqrt{6}-\sqrt{2}\right)^2 = 6-2\sqrt{12}+2$ ***(A1)***

$\sqrt{24-4\sqrt{3}}$ ***A1***

Note: The expression for the form of the value of r in the question uses a "+" sign but the answer has a "–" sign. This may cause some confusion

[3 marks]

(b) $\tan^{-1}\left(\frac{\sqrt{6}-\sqrt{2}}{-4}\right)$ **OR** $-0.253261...$ **OR** $-14.5108...$ (from degrees) seen ***(A1)***

$(\theta =)2.89\ (2.88833...)$ ***A1***

Note: Accept $\sin^{-1}\left(\frac{\sqrt{6}-\sqrt{2}}{\sqrt{24-4\sqrt{3}}}\right)$ or $\cos^{-1}\left(\frac{-4}{\sqrt{24-4\sqrt{3}}}\right)$ with ***FT*** from part (a) if necessary

[2 marks]

(c) stretch by a factor $\left(\sqrt{24-4\sqrt{3}}\right)^2$ or $(4.13180...)^2$ or $17.0717...$ ***R1***

(anticlockwise) rotation of $2\times 2.89\,(2.88833...)$ or $5.78\,(5.77666...)$ radians

OR (clockwise) rotation of $0.507\ (0.506525\ldots)$ ***R1***

Note: Award ***R1R0*** for responses that state that there is a stretch and a rotation without the correct values. Condone ***R1*** for enlargement instead of stretch

[2 marks]

Total *[7 marks]*

14. (a) (i) $\frac{1}{2}\left(\frac{2}{4}, 0.5\right)$ ***A1***

(ii) **METHOD 1**

attempt to find value of tangent for larger triangle ***M1***

$\frac{r}{x+4}$

$\frac{r}{x+4}=\frac{1}{2}$

$2r=x+4$ ***A1***

$x=2r-4$ ***AG***

METHOD 2

any correct proportion seen ***M1***

$\frac{4}{2}=\frac{x+4}{r}$ **OR** $\frac{r}{2}=\frac{x+4}{4}$

correct algebra leading to given answer ***A1***

$4r=2(x+4)$ **OR** $2r=x+4$

$x=2r-4$ ***AG***

[3 marks]

(b) attempt to use volume of cone formula **AND** subtract volume of removed cone ***(M1)***

$V=\frac{1}{3}\pi r^2(4+x)-\frac{1}{3}\pi(2)^2(4)$

substitution of expression $x=2r-4$ from part (a) into their equation ***(M1)***

$V=\frac{1}{3}\pi r^2(4+2r-4)-\frac{1}{3}\pi(2)^2(4)$

$V=\frac{2}{3}\pi r^3-\frac{16}{3}\pi$

$(a=)5.33\left(5.33333..., \frac{16}{3}\right)$ ***A1***

Note: Award ***M0M1A0*** for substituting expression $x=2r-4$ from part (a) into the equation of a cone that has not had a cone removed.

[3 marks]

continued…

Question 14 continued

(c) $5 = 2\pi r^2 (0.06)$ ***(A1)***

attempt to solve for r ***(M1)***

$r = \pm 3.64182...$

correctly substitute their positive r into $x = 2r - 4$ ***(M1)***

$x = 3.28 \ (3.28365...)$ cm ***A1***

[4 marks]

Total [10 marks]

15. (a) attempt to solve the characteristic equation ***(M1)***

$$\det\begin{pmatrix} 2-\lambda & -5 \\ 1 & -1-\lambda \end{pmatrix} = 0$$

$$(2-\lambda)(-1-\lambda)+5(=0) \quad \textit{(A1)}$$

$$\lambda^2 - \lambda + 3(=0) \quad \textit{(A1)}$$

$0.5 \pm 1.66\text{i}$ $\left(0.5 \pm 1.65831...\text{i}, \dfrac{1 \pm \text{i}\sqrt{11}}{2}, \dfrac{1}{2} \pm \dfrac{\sqrt{11}}{2}\text{i}\right)$ ***A1***

[4 marks]

(b) rotational or spiraling because complex eigenvalues ***R1***

outwards movement because real part of both eigenvalues is positive ***R1***

substituting $(1, 0)$ into system to find $\dot{y} = 1$ therefore anticlockwise ***R1***

anti-clockwise spiral outwards from the origin ***A1***

Note: ***A1*** requires all three ***R1***'s.
Award the ***R1***'s independently.

[4 marks]

Total ***[8 marks]***